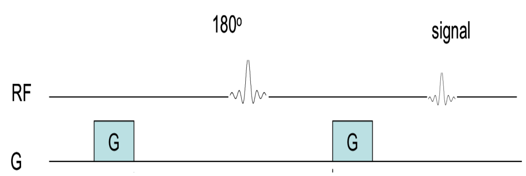
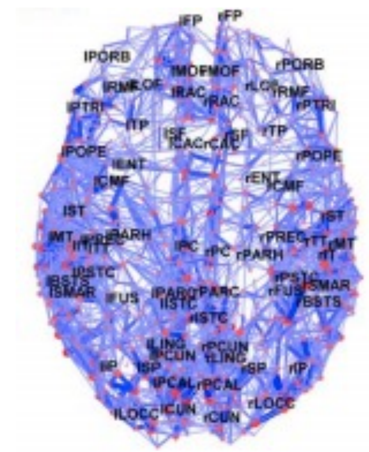
Diffusion MR Imaging





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Biomedical Engineering, Emory University & GaTech
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Outline

- Basics of diffusion, diffusion in tissue, measurement of diffusion using with MR
- Diffusion Parameters and Tractography
- Current Clinical Applications
- Connectomics, Advanced Diffusion Methods and Models

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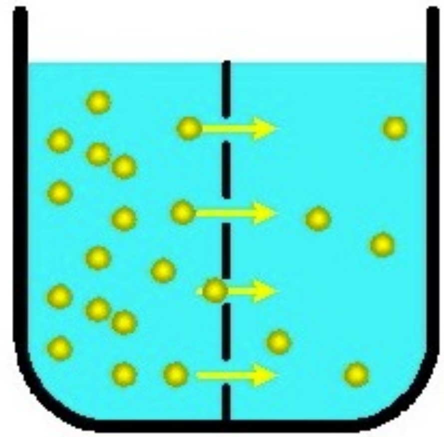
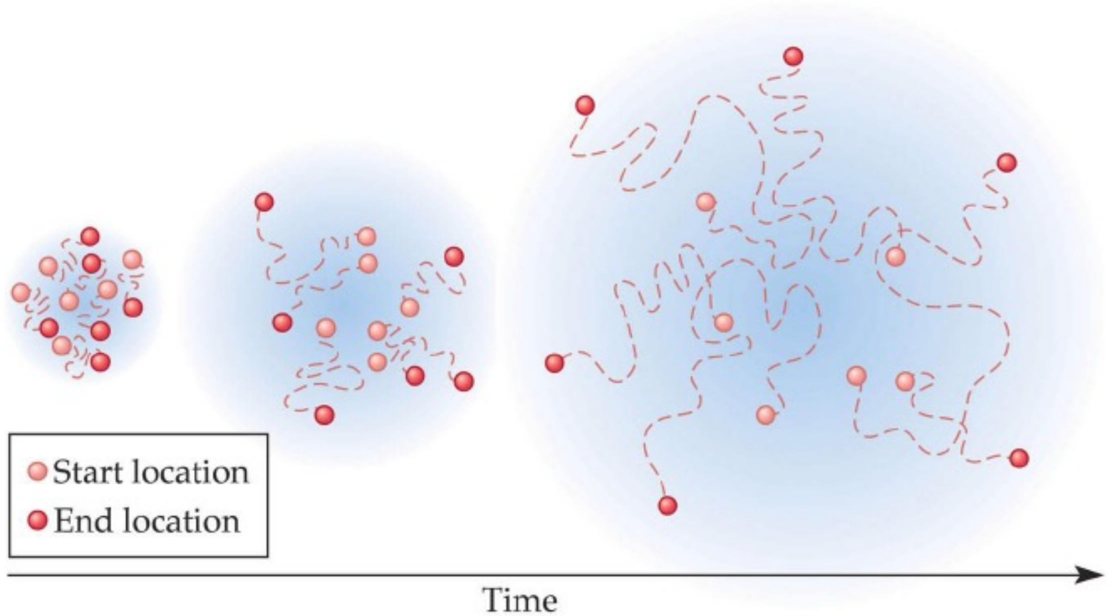
Macroscopic Diffusion

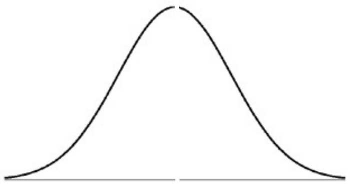


$$J = -D \cdot \nabla \cdot n(\vec{r}, t)$$

Fick's Law

Microscopic/self diffusion





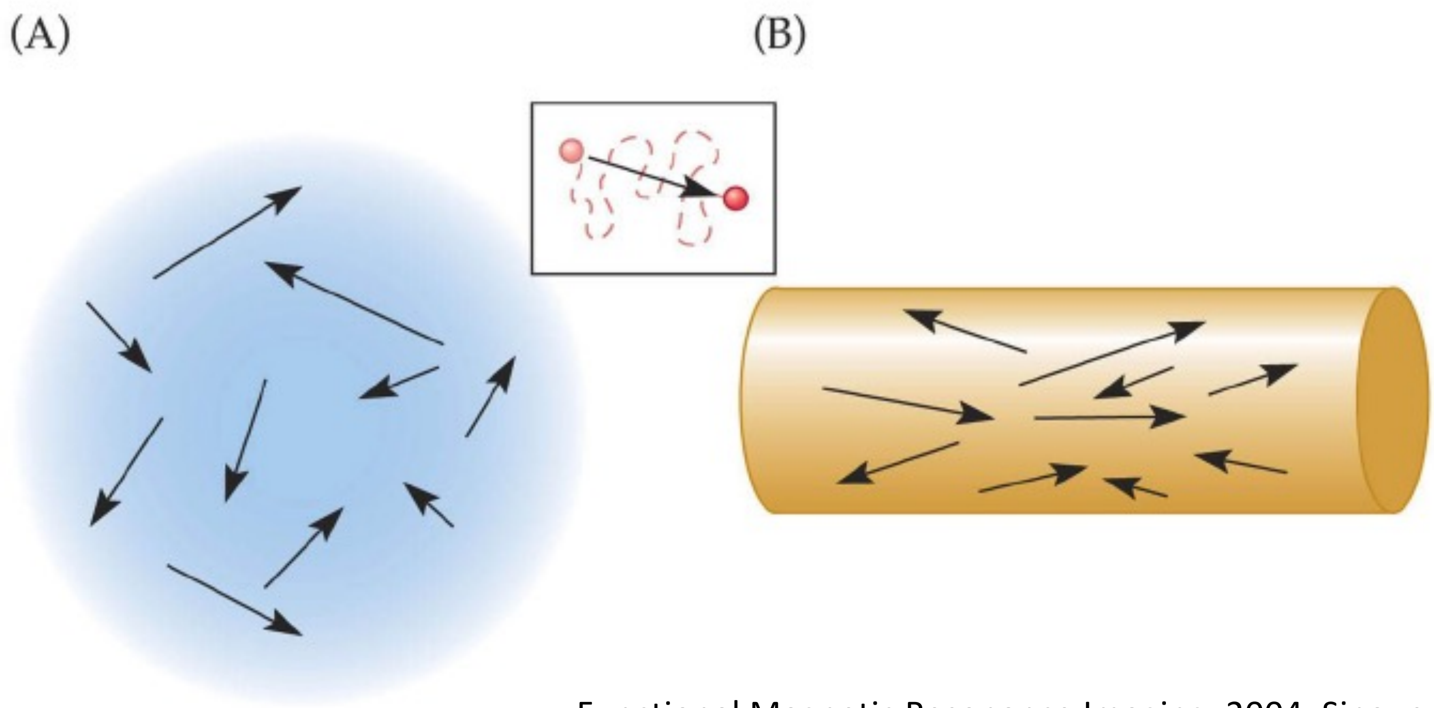
- Einstein Equation:

$$RMS = \sqrt{2D \cdot t}$$

D=2.6x10^{-3} mm2/s
@ 25^{o}

Functional Magnetic Resonance Imaging. 2004. Sinauer Associates Inc

Isotropic vs Anisotropic diffusion



Functional Magnetic Resonance Imaging. 2004. Sinauer Associates Inc

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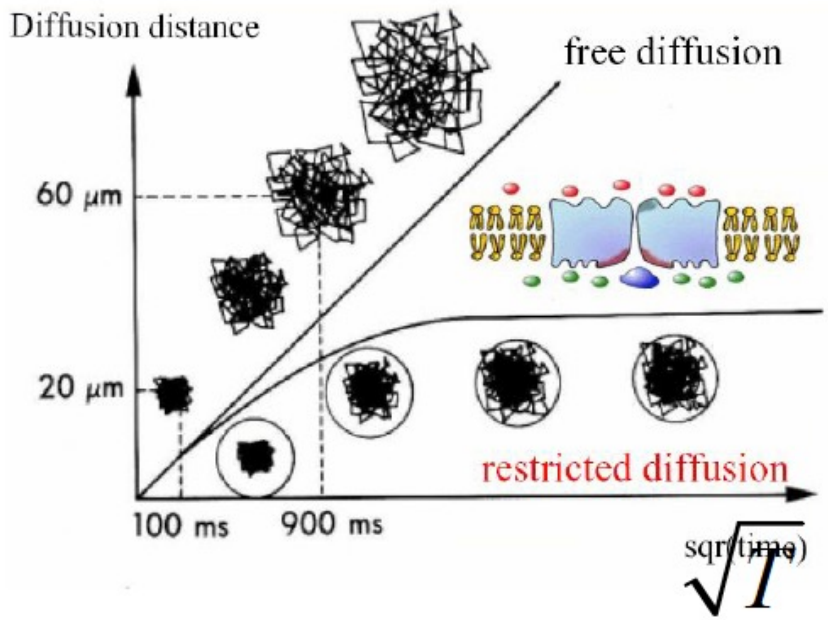
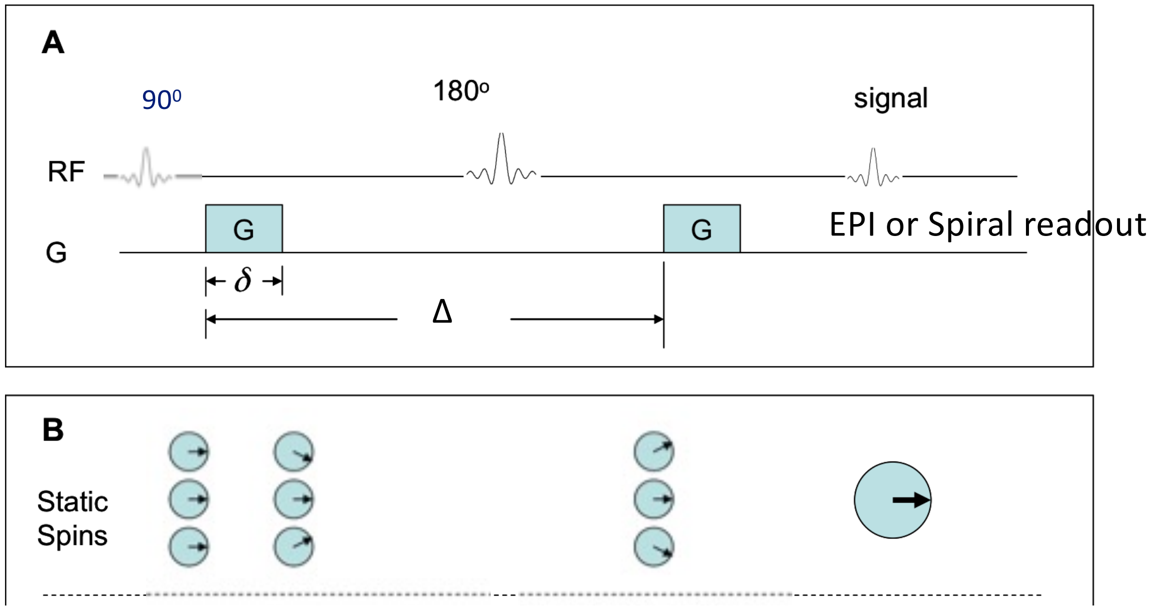
Free Diffusion vs Restricted Diffusion

- Free or unhindered Diffusion
  - RMS ~ sqrt(Dt)
  - ~3um^2/ms –Body temperature in water (CSF)
- Restricted or hindered diffusion
  - **<u>Apparent</u>** Diffusion Coefficient (ADC)



Stejskal-Tanner Sequence for Diffusion



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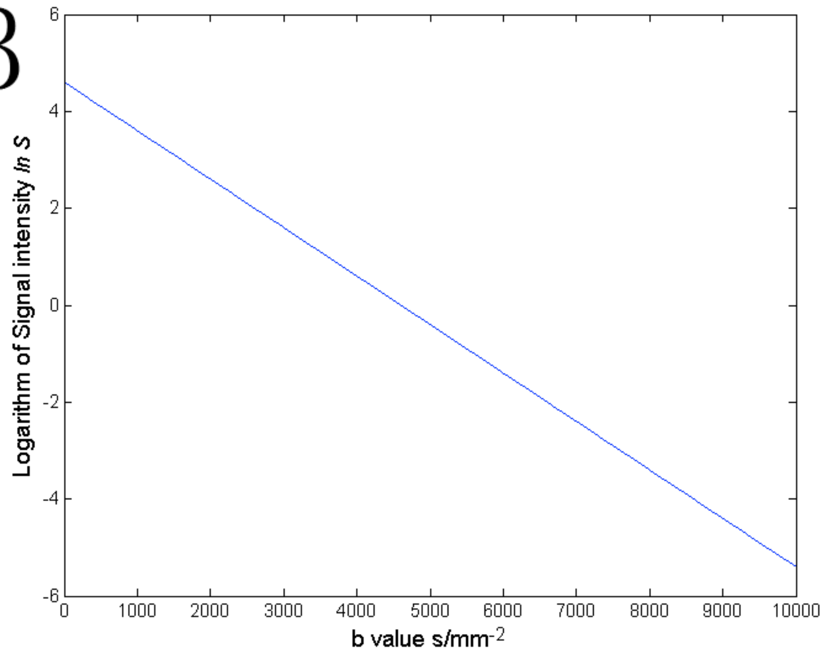
Single Direction Diffusion Signal Model

B



$$S = S_0 \exp(-b \cdot D)$$

$$b = \gamma^2 G^2 \delta^2 \left(\Delta - \frac{\delta}{3}\right)$$

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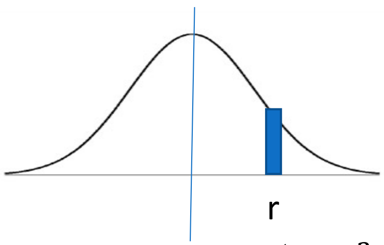
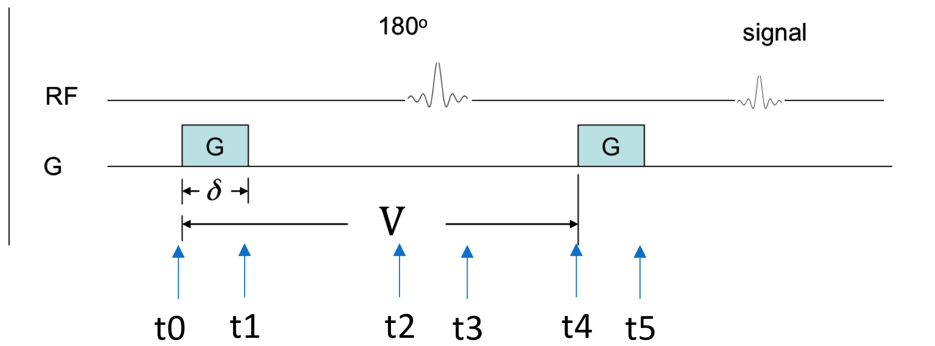
Deriving diffusion weighting equation



$$P(r) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\sigma = \sqrt{2D \cdot \Delta}$$

$$\varphi(r) = \gamma G\delta r \coloneqq qr$$



$\varphi(t0) = 0$

$\varphi(t1) = \gamma B(k)t = \gamma G\mathrm{k}\delta = \gamma G\delta\mathrm{k}$, for spin at an initial location k

$\varphi(t2) = \varphi(t1) = \gamma G\delta\mathrm{k}$

$\varphi(t3) = -\varphi(t2) = -\gamma G\delta\mathrm{k}$

$\varphi(t4) = \varphi(t3) = -\gamma G\delta\mathrm{k}$

Spin has moved to location k+r

$\varphi(t5) = \varphi(t4) + \gamma G\delta(\mathrm{k+r}) = \gamma G\delta\mathrm{r}$

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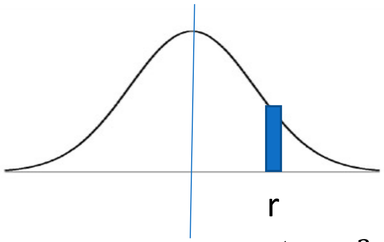
Deriving diffusion weighting equation



$$P(r) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\sigma = \sqrt{2D \cdot \Delta}$$

$$\varphi(r) = \gamma G \delta r \coloneqq qr$$

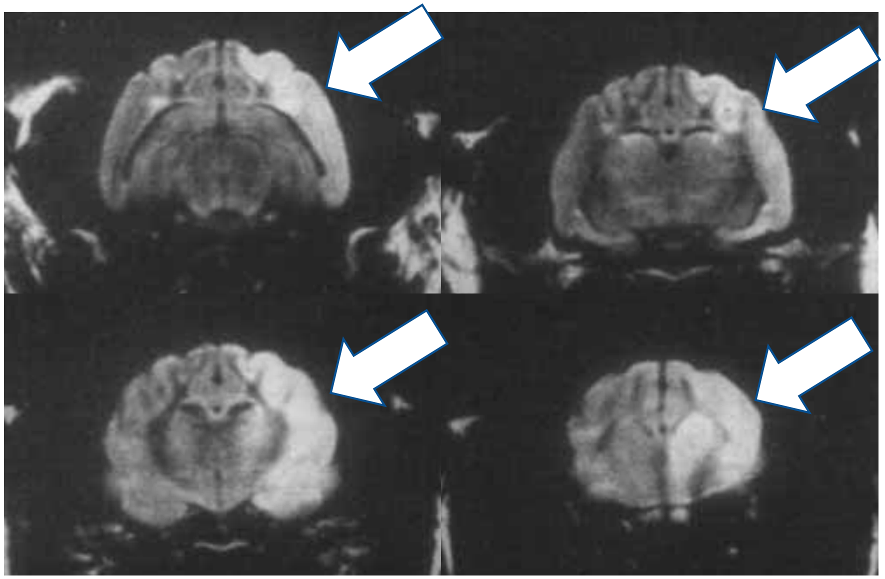
$$S = S_0 \cdot \int P(r) \cdot \exp\big(i\varphi(r)\big)\, dr$$

$$= \int \frac{\exp\left(-\frac{r^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma} \exp(iqr) dr$$

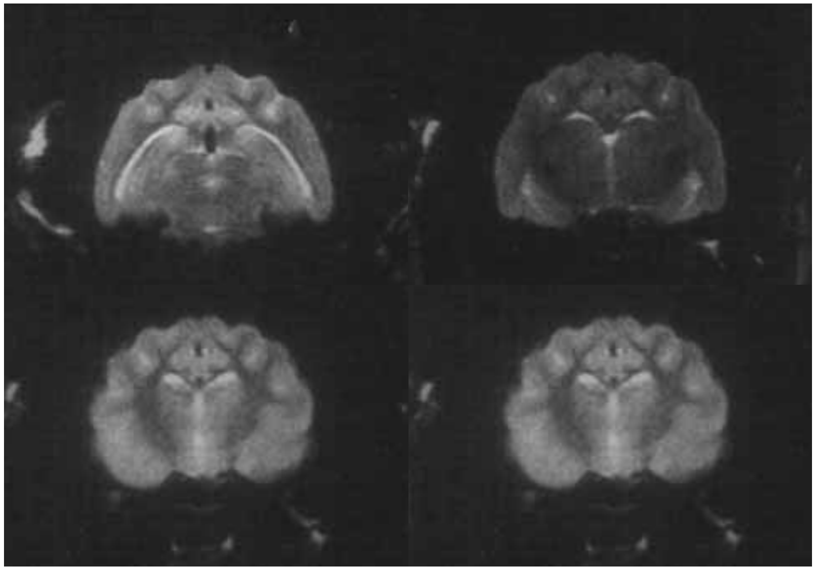
$$= \quad \exp\left(-\frac{1}{2}\sigma^2 q^2\right) \quad = \exp(-\gamma^2 G^2 \delta^2 \Delta D)$$

General formula: $b = \gamma^2 \int_0^{TE} \left(\int_0^t G(t') \cdot Sign(t' - \frac{TE}{2}) dt'\right)^2 dt$

Early Detection of Stroke with DWI



DWI taken 45mins after experimental occlusion



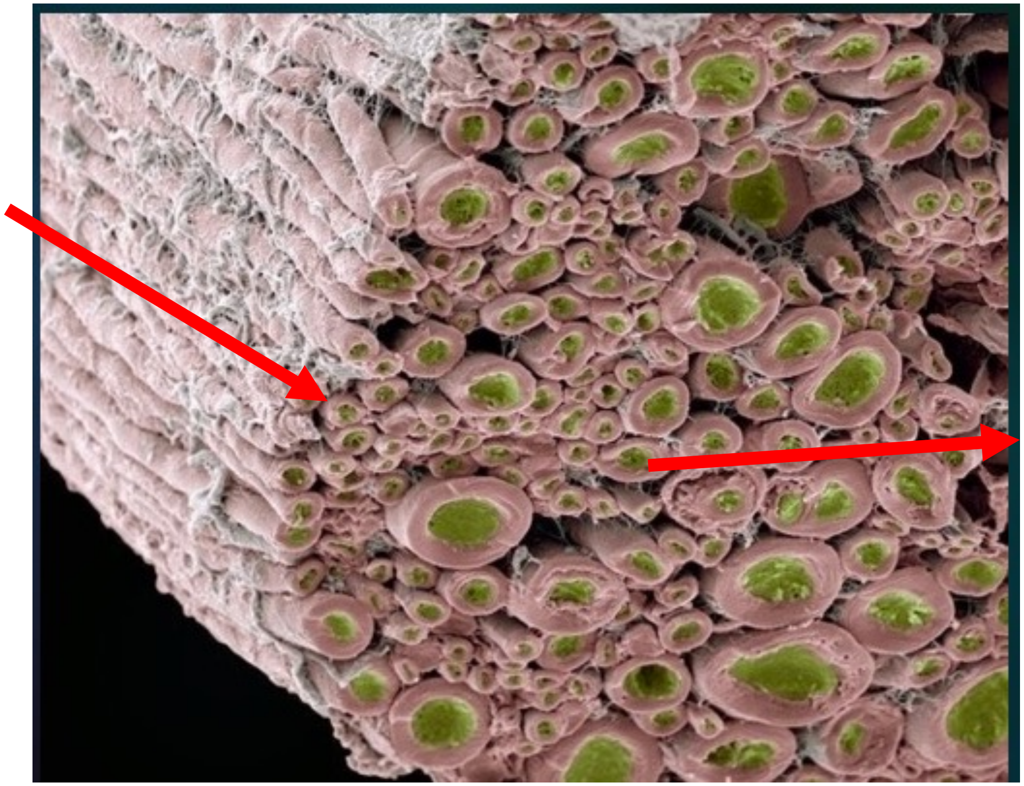
T2w taken 65mins after experimental occlusion

Moseley M et al. 1990. MRM

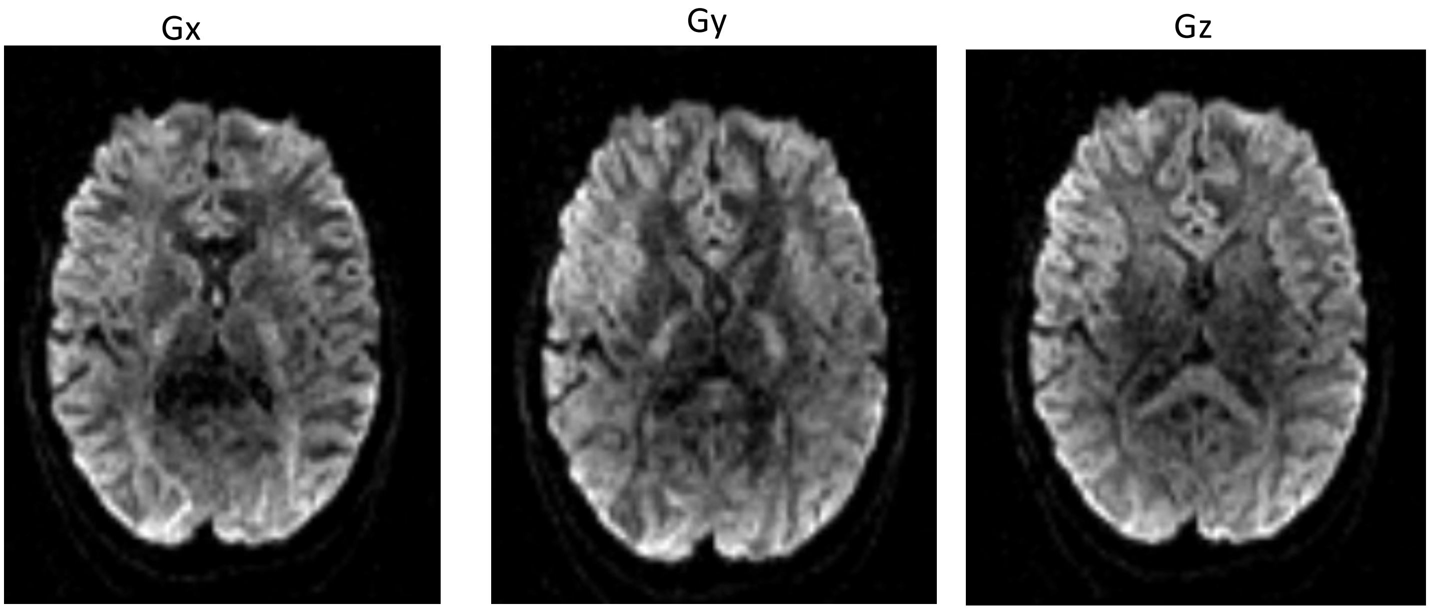
Application of DWI in Stroke

MGH

Modeling Anisotropic Diffusion



DWI



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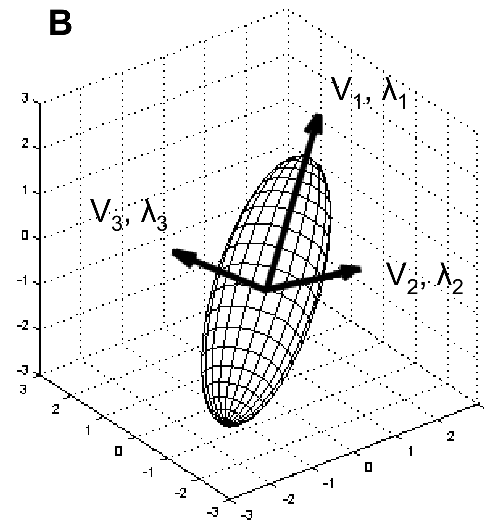
Diffusion Tensor Imaging

- Describe the diffusion process with a 3x3 Matrix (tensor)
- It is a covariance matrix of diffusion probability distribution
- Ellipsoid defines a surface of constant mean translational displacement



$$P(r) = \frac{1}{(2\pi)^{3/2}\Sigma^{1/2}} \exp\left(-\frac{1}{2} r^T \Sigma^{-1} r\right)$$

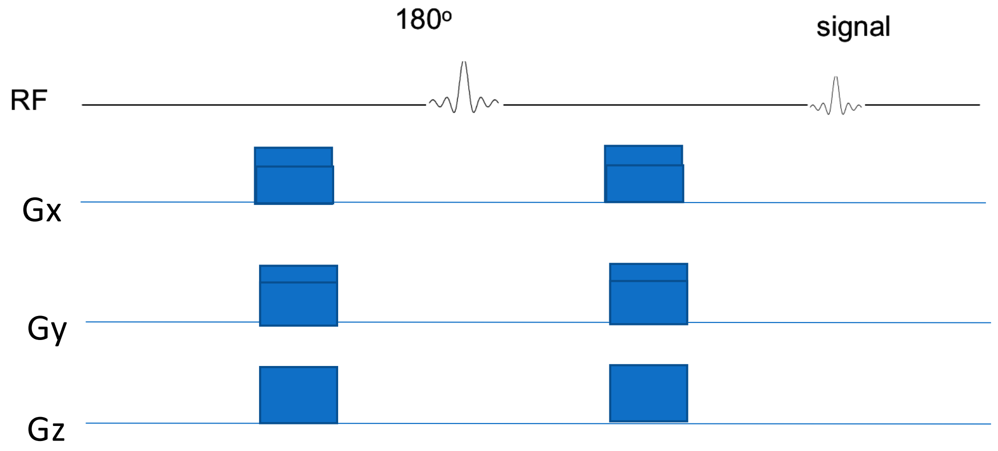
$$\Sigma = 2D_{eff} \cdot \Delta$$

$$\mathbf{D}_{eff} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

Measuring diffusion in multiple directions

$$S(\vec{v}_i)_{i=1,\ldots n} = S_0 \cdot \exp(-b * \vec{v}_i^{\,T} \cdot D_{eff} \cdot \vec{v}_i)$$

$$\ln(S(\vec{v}_i))_{i=1,\ldots n} = \ln(S_0) - b * \vec{v}_i^{\,T} \cdot D_{eff} \cdot \vec{v}_i$$



1, 0, 0

0.707, 0.707, 0

0, 1, 0

0, 0, 1

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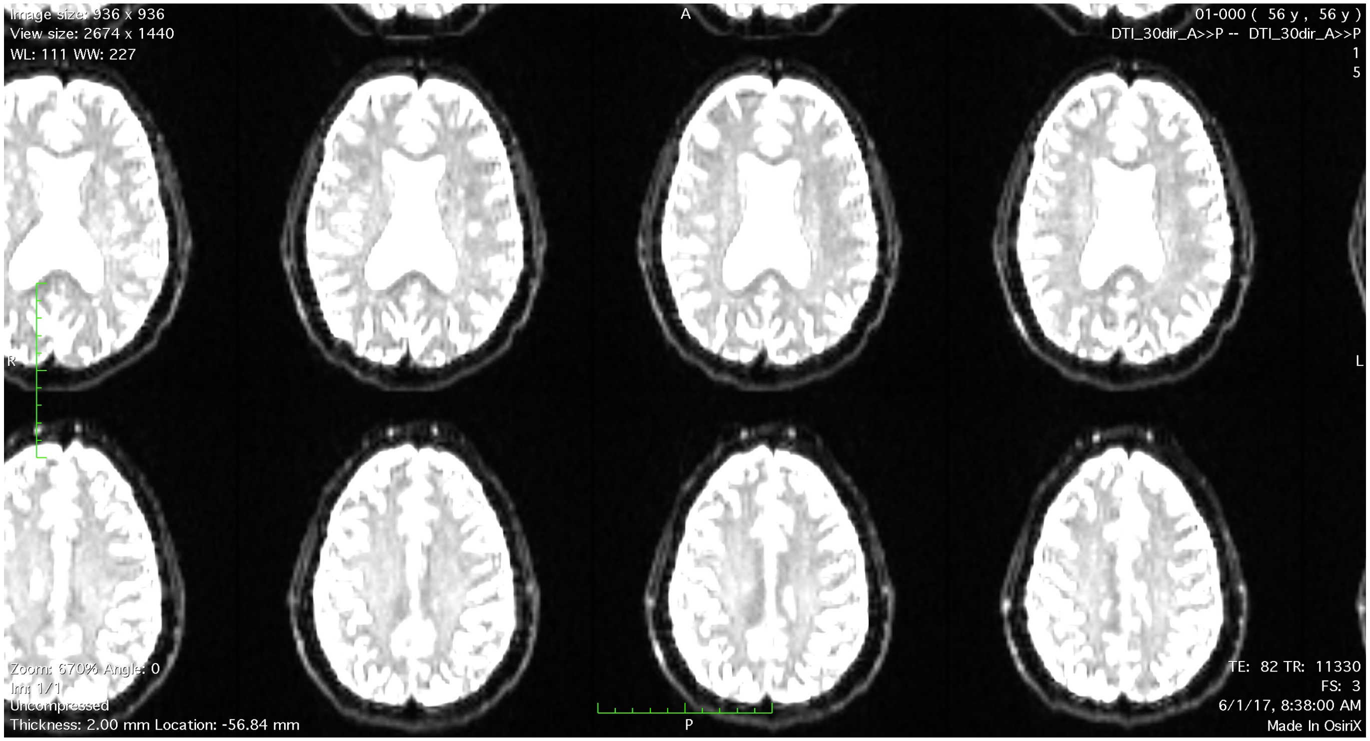
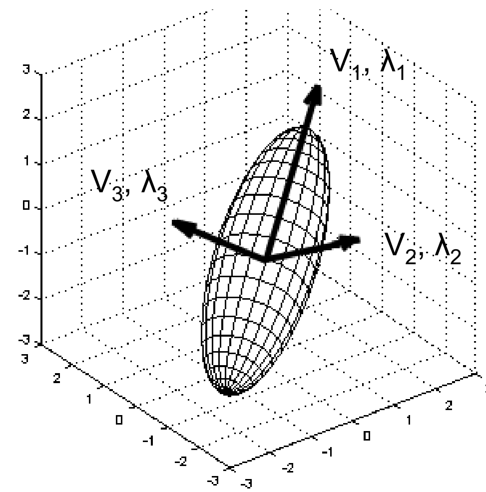
Measuring diffusion in multiple directions

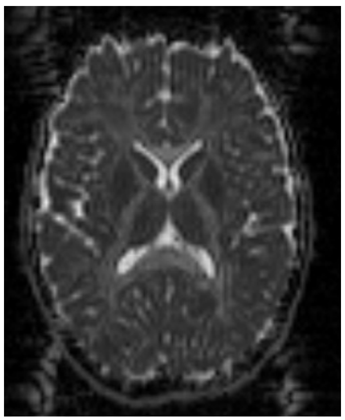


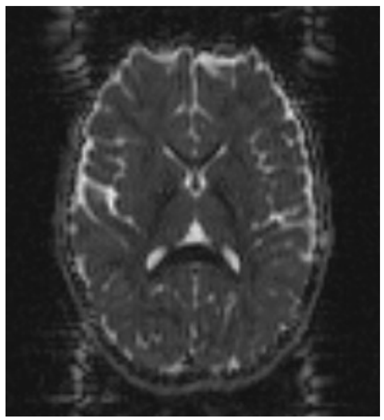
Diagonalization of the Tensor

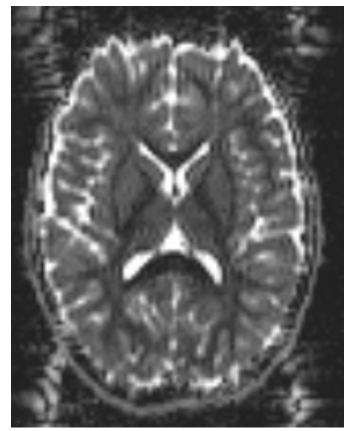
$$\mathbf{D}_{eff} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

$$D = [V_1, V_2, V_3] \cdot \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} \cdot [V_1, V_2, V_3]^T$$




λ1


λ2


λ3

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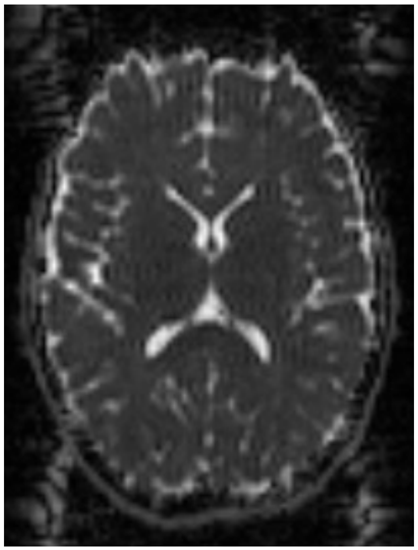
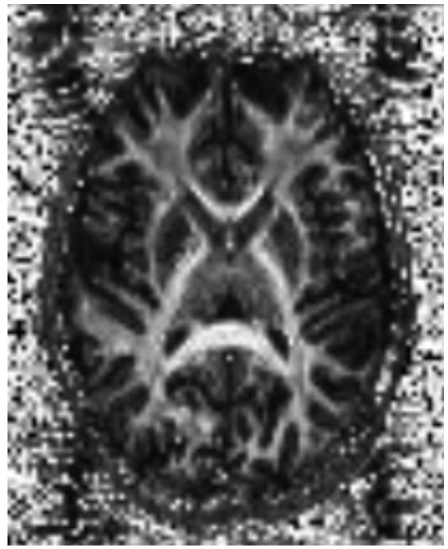
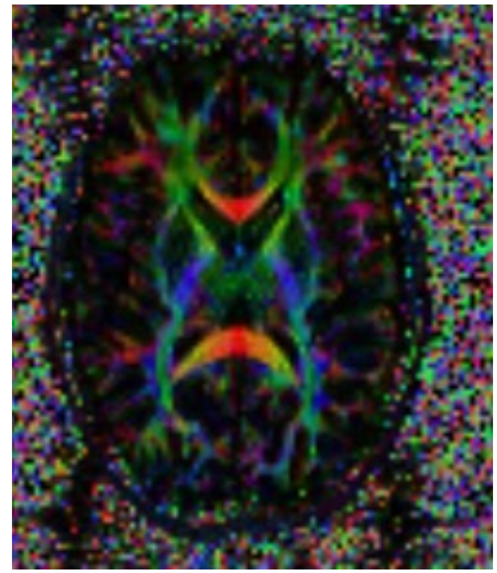
Commonly Used Metrics and Demo



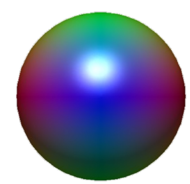




$$MD = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

$$FA = \sqrt{\frac{3}{2} \cdot \frac{(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

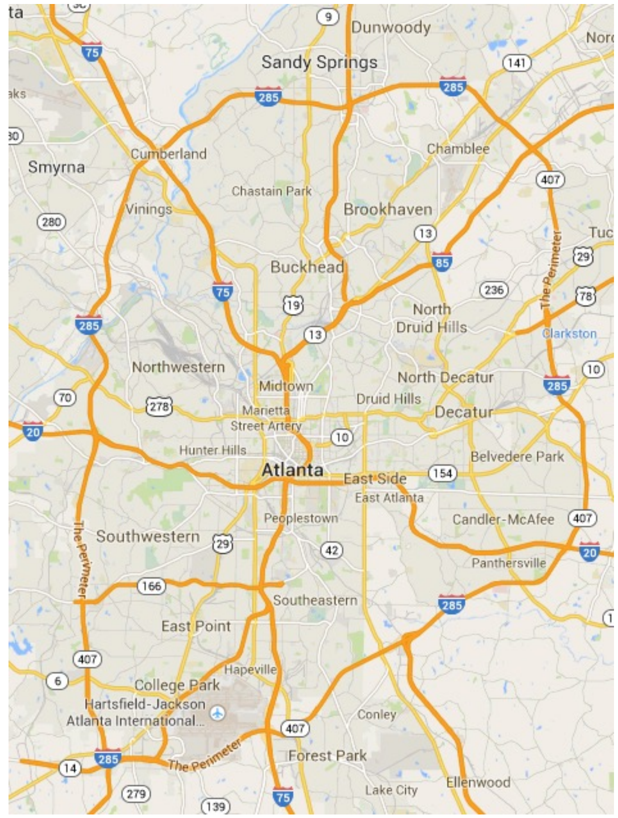
Color FA



Demo codes

- https://randomprogram.net/software/Slides/DTI-processing-codes.zip

Wiring of the Brain



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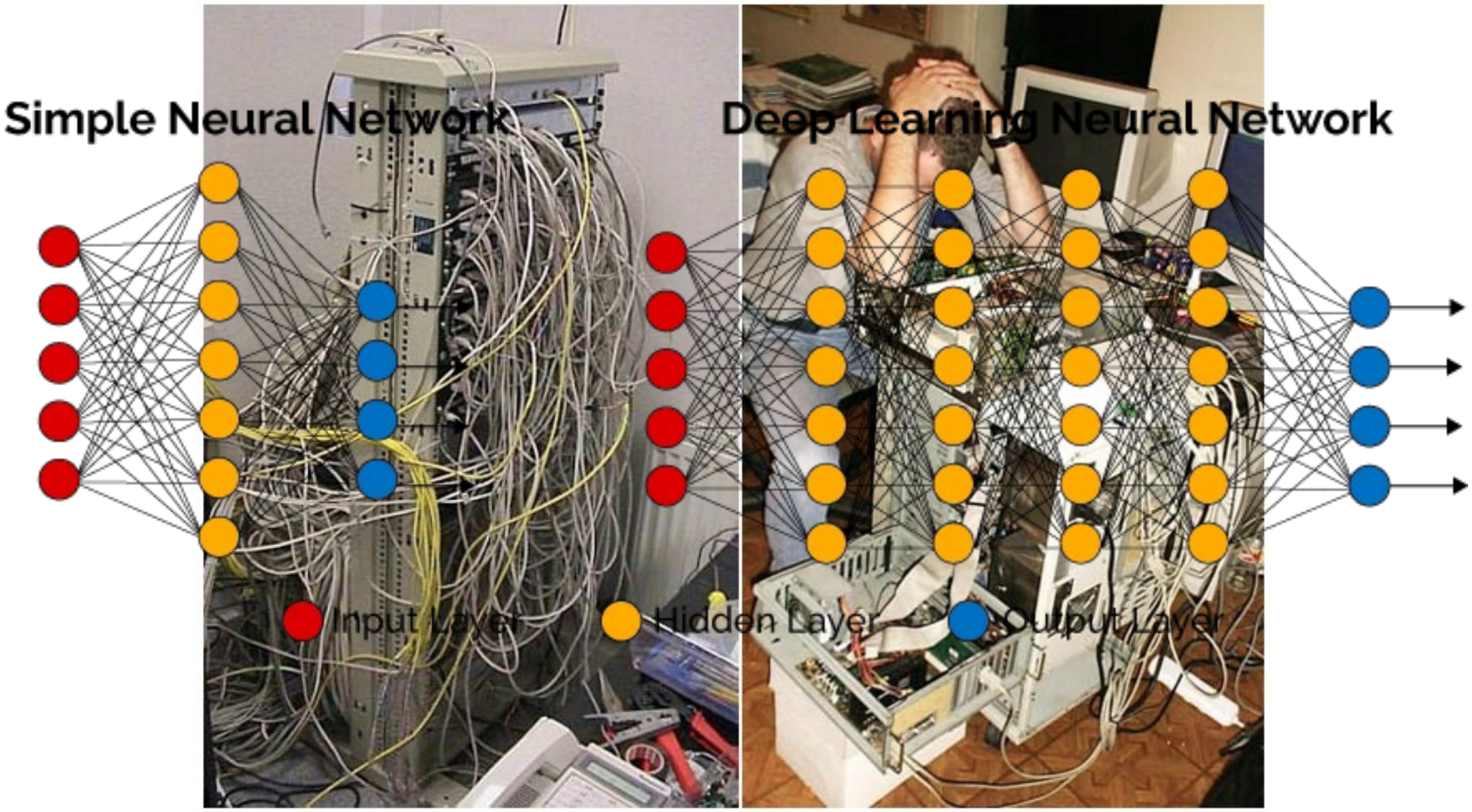
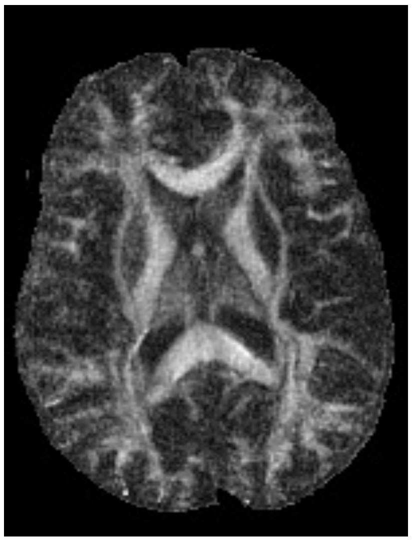
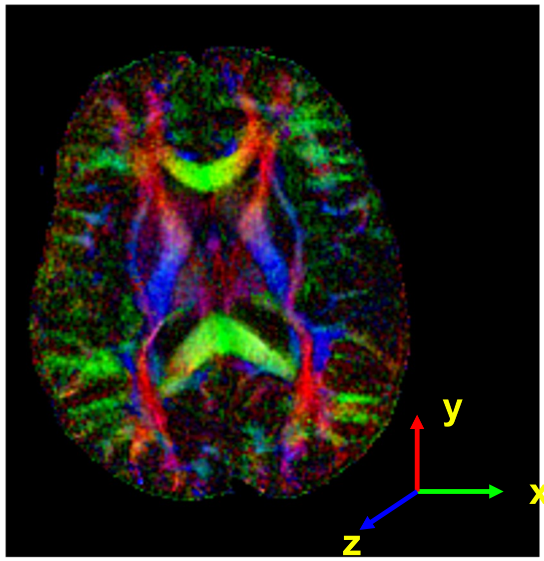
It’s all about how things are wired



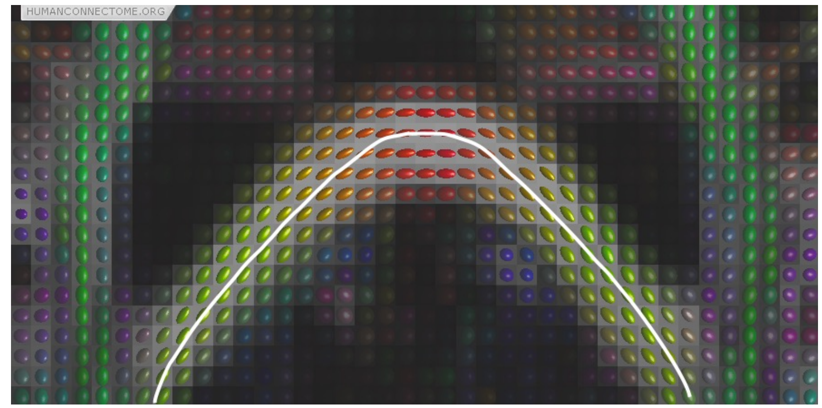
How Do We Show DTI: Tractography!

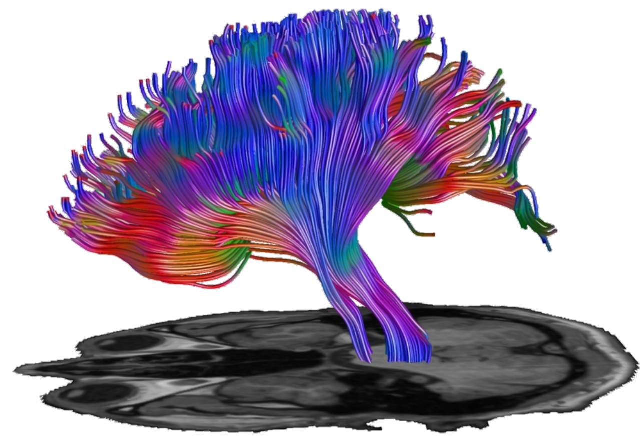


DTI “FA” map



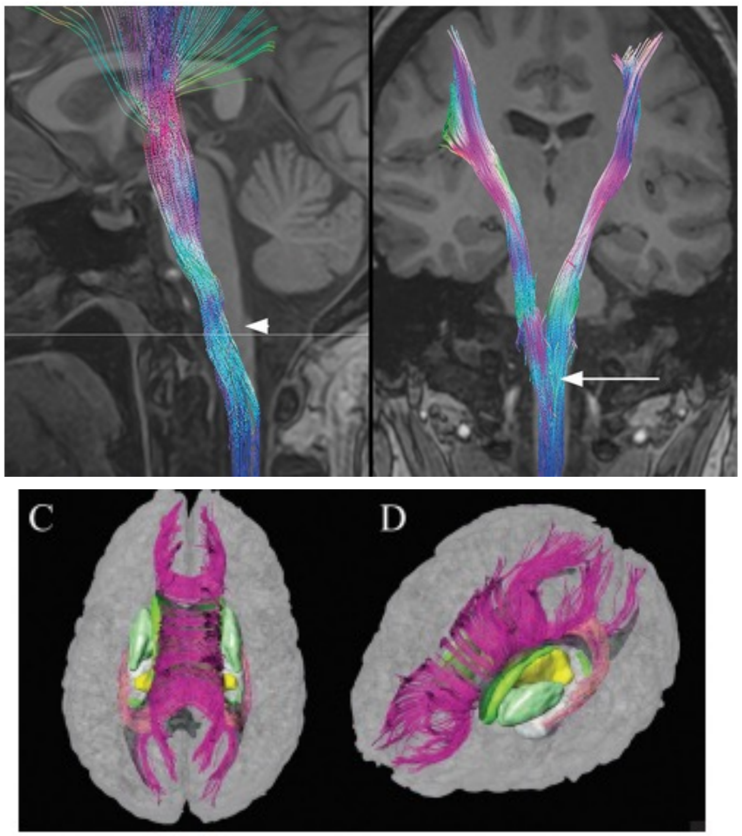
Color“FA” map

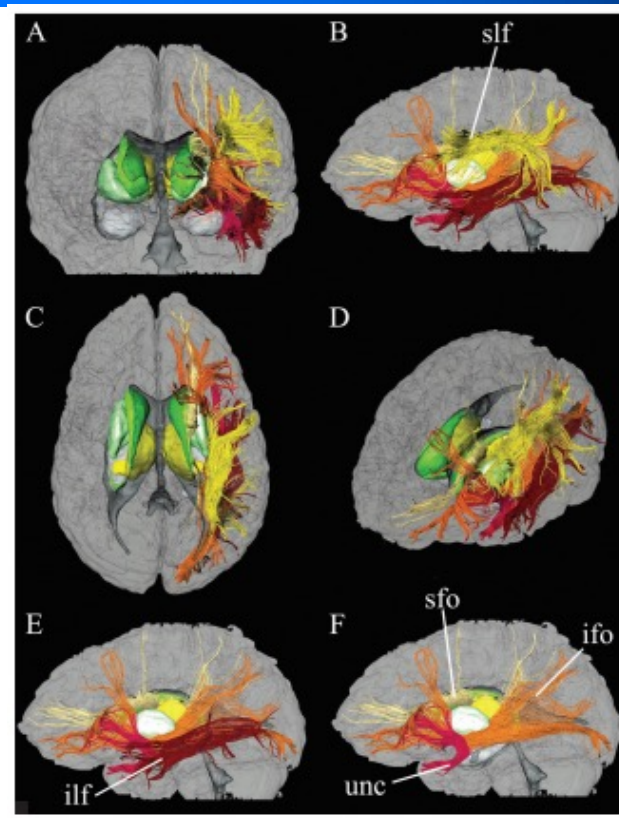




Fiber Tractography

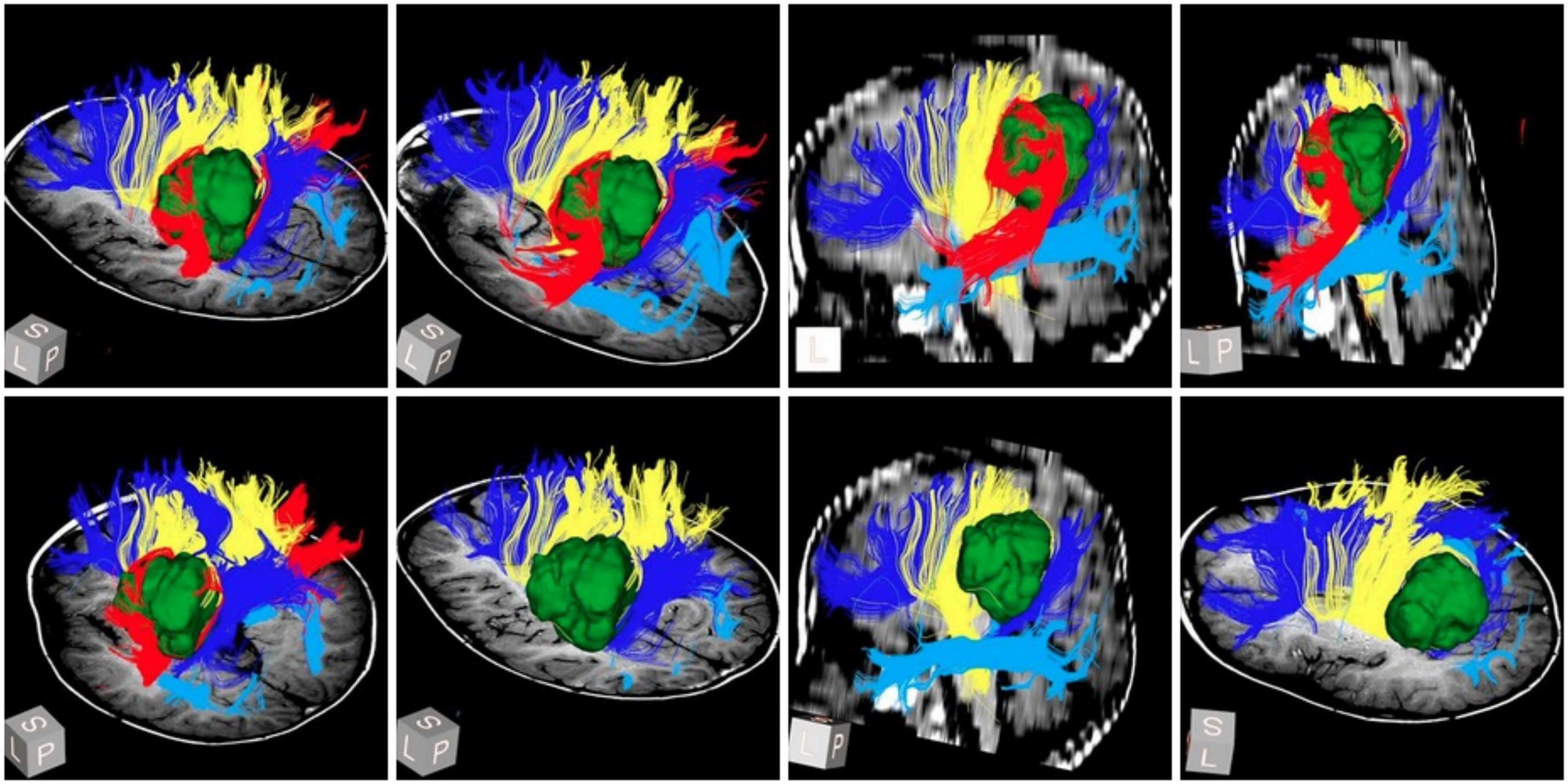
Major Fiber Systems in the Brain





Wakana S. et al. 2003. Radiology

Tractography in Surgery Planning



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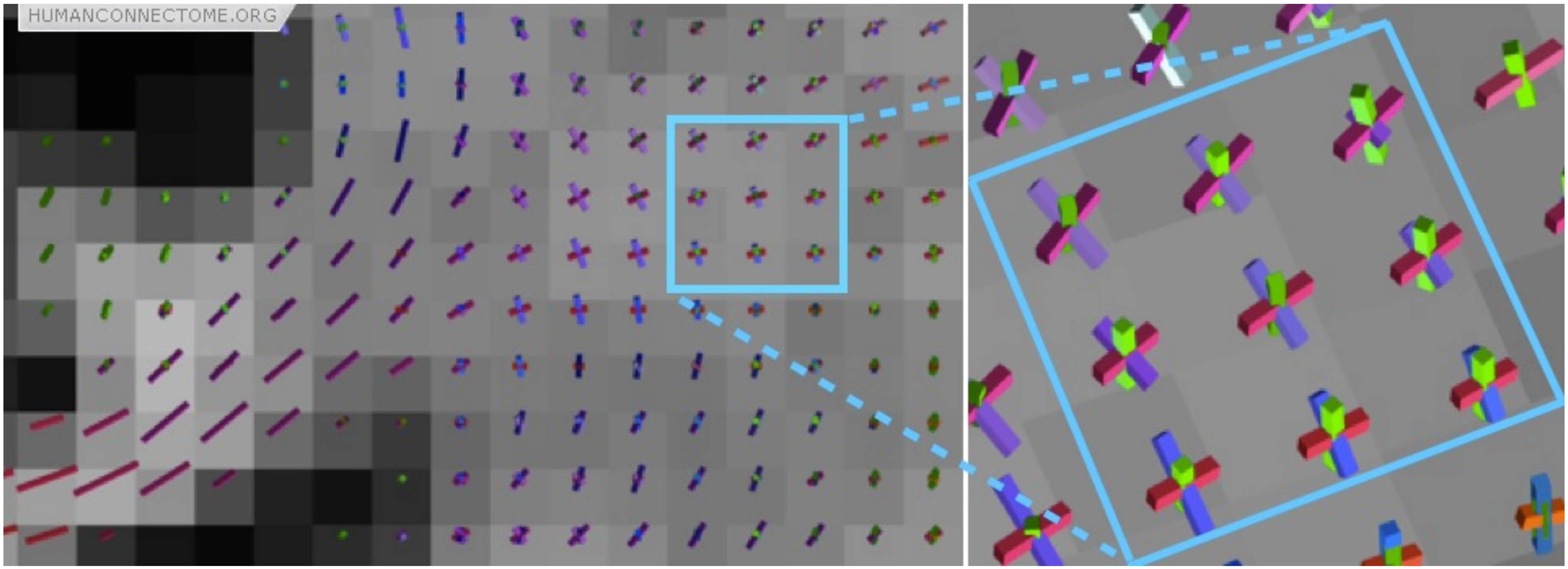
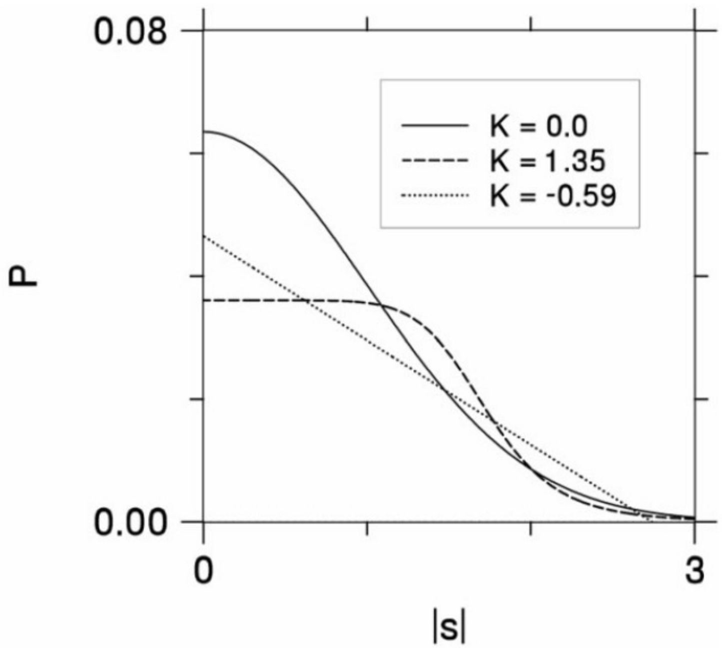
Crossing Fiber Issue



Pushing to higher order: Diffusion Kurtosis Imaging



$$\ln\,[S(b)] = \ln\,[S(0)] - bD_{\text{app}} + \frac{1}{6}\,b^2 D_{\text{app}}^2 K_{\text{app}} + O(b^3),$$

Jensen JH. et al. 2005. MRM

Mean Diffusivity vs Kurtosis

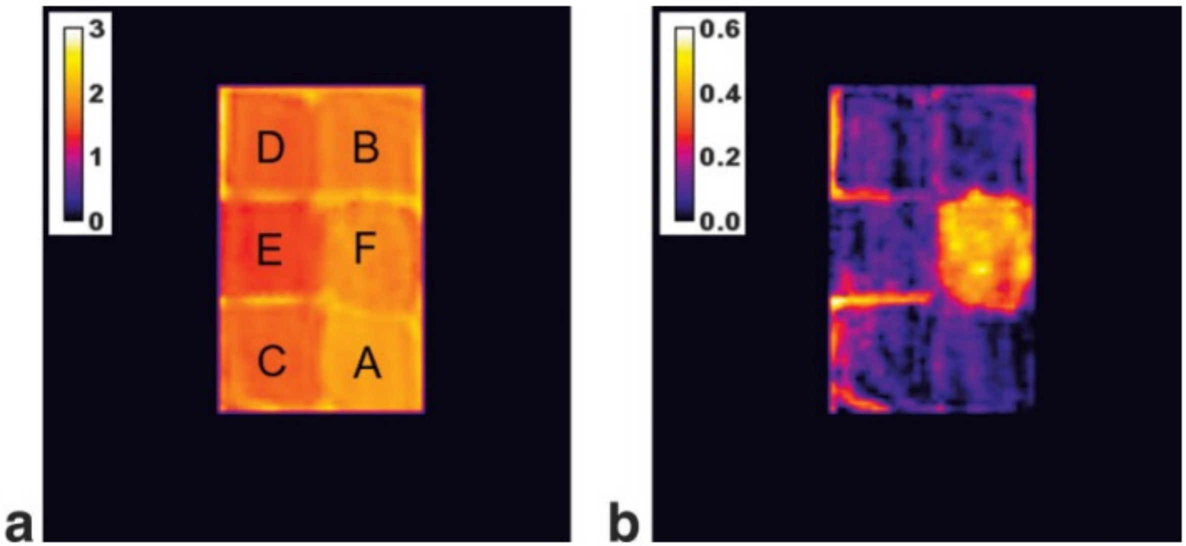


FIG. 5. Parametric maps of the apparent diffusion coefficient (left) and the apparent diffusional kurtosis (right) for the phantom in the slice direction. The scale bar for the diffusion coefficient is in units of $\mu m^2/ms$. Bottles a through e contain sucrose solutions with sucrose concentrations ranging from 5 to 25%. Bottle f contains pureed asparagus. The average kurtosis map clearly reveals the higher degree of structure in asparagus bottle, which is not evident in the diffusion coefficient map.

Jensen JH. et al. 2005. MRM

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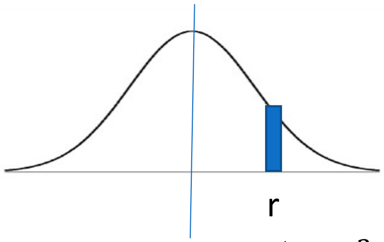
Deriving diffusion weighting equation



$$P(r)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\sigma=\sqrt{2D\cdot\Delta}$$

$$\varphi(r)=\gamma G\delta r\coloneqq qr$$

$$S=S_0\cdot\int P(r)\cdot\exp\big(i\varphi(r)\big)\,dr$$

$$=\int\frac{\exp\left(-\frac{r^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}\exp(iqr)dr$$

$$=\int\frac{\exp\left(-\frac{r^2}{2\sigma^2}+iqr\right)}{\sqrt{2\pi}\sigma}dr$$

$$=\int\frac{\exp(-\frac{(r-iq\sigma^2)^2}{2\sigma^2}-\frac{q^2\sigma^4}{2\sigma^2})}{\sqrt{2\pi}\sigma}dr$$

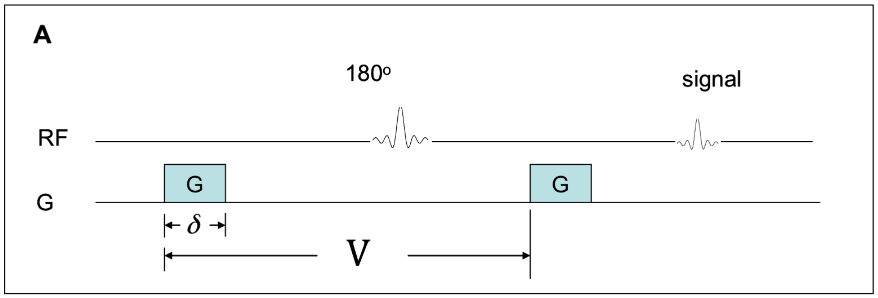
$$=\exp(-\frac{q^2\sigma^2}{2})\int\frac{\exp(-\frac{(r-iq\sigma^2)^2}{2\sigma^2})}{\sqrt{2\pi}\sigma}dr$$

$$=\exp(-\frac{q^2\sigma^2}{2})\int_{-\infty+iq\sigma^2}^{+\infty+iq\sigma^2}\frac{\exp(-\frac{r^2}{2\sigma^2})}{\sqrt{2\pi}\sigma}dr$$

$$=\exp(-\gamma^2G^2\delta^2\Delta D)$$

$$b=\gamma^2\int_0^{TE}\left(\int_0^t G(t')\cdot Sign(t'-\frac{TE}{2})dt'\right)^2dt$$

Q-Space Imaging: 1-D



q= $\gamma G \delta$

S(q)= $\int \exp(iqx) \cdot p(x)dx$

Diffusion Propagator p

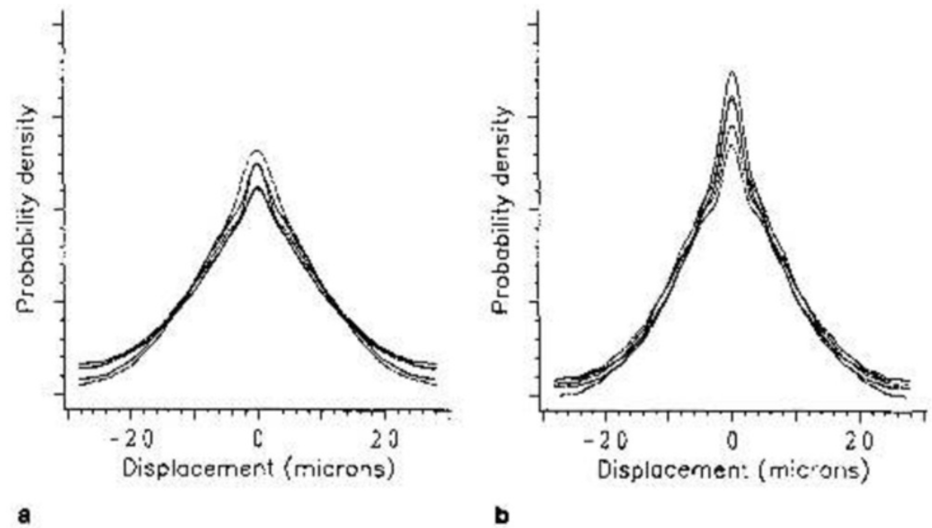
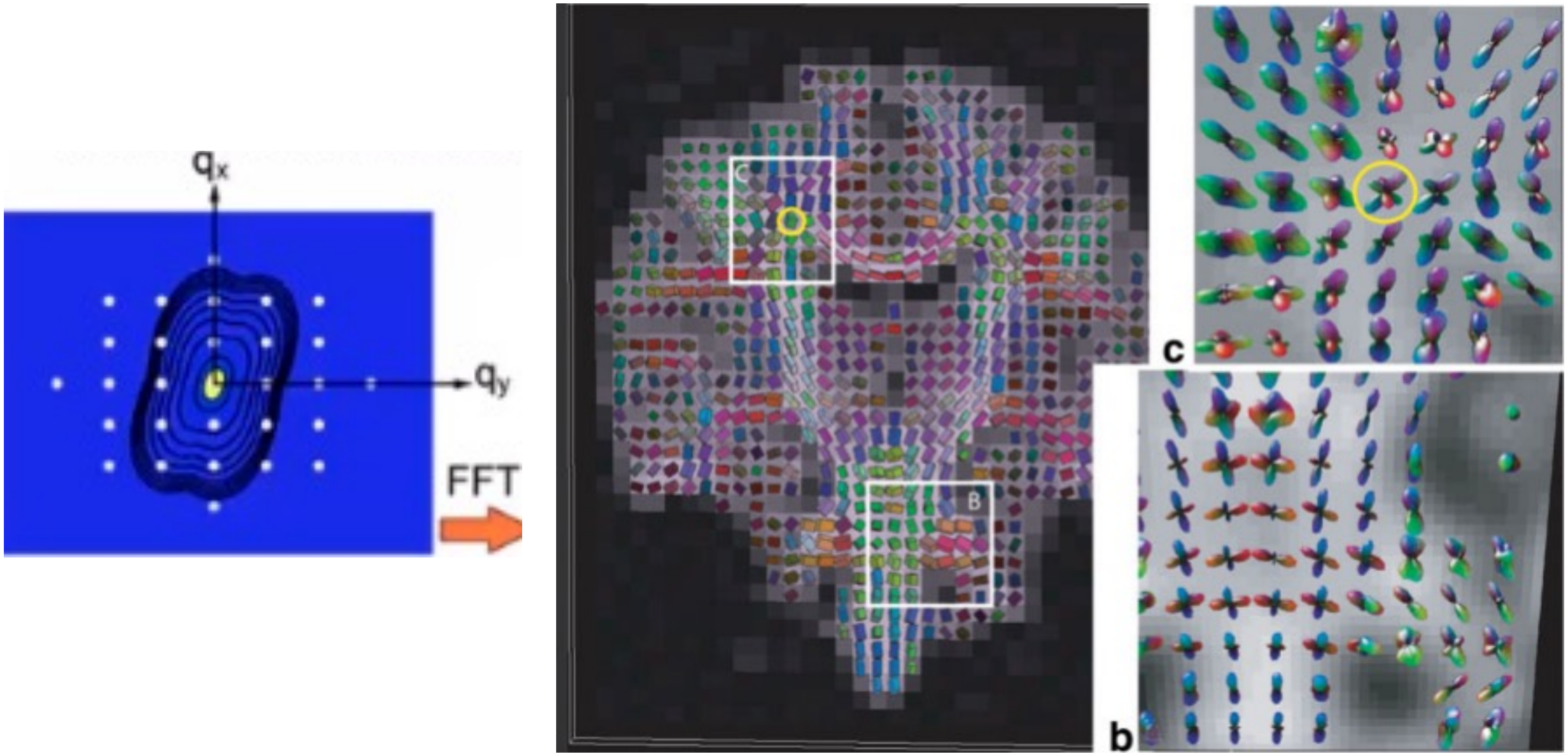


Fig. 2. Brain water displacement profiles. The profiles obtained from five normal animals are shown Fig. 2a, and those acquired 1 h *postmortem* in Fig. 2b. The diffusion period was 101 ms in duration.

King M. 1994. MRM

Diffusion Spectrum Imaging



Wedeen V. 2005. Neuroimage

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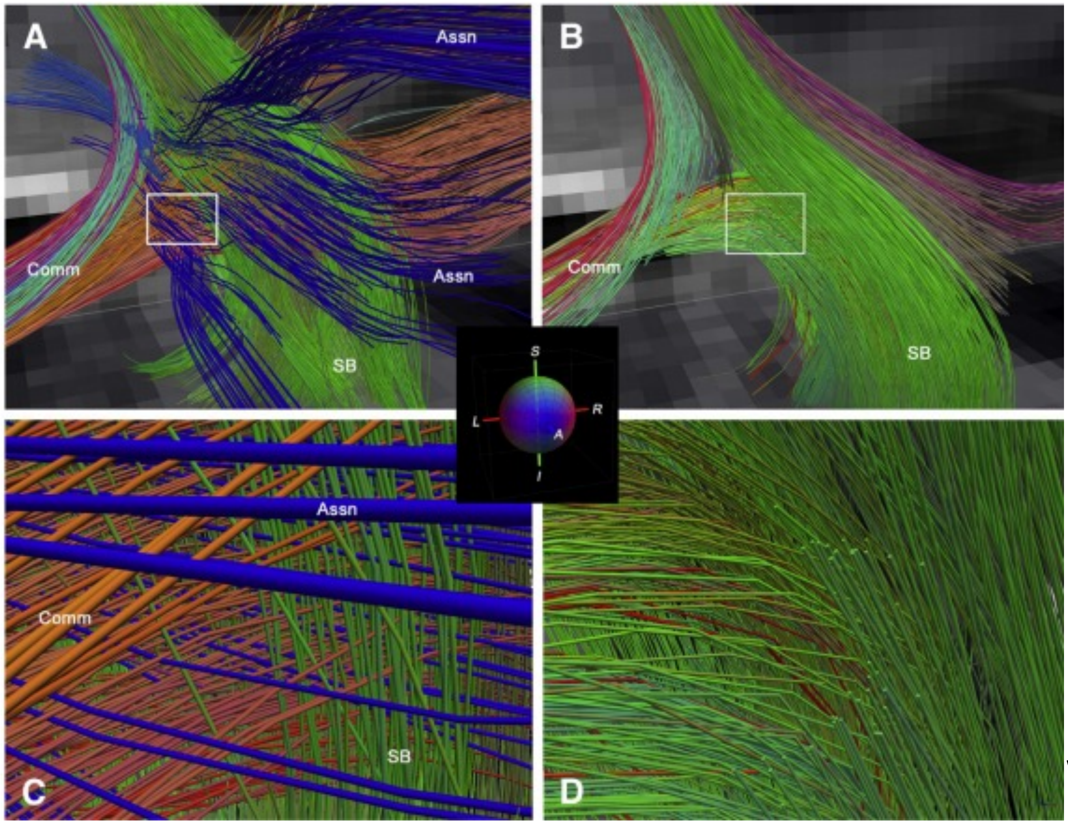
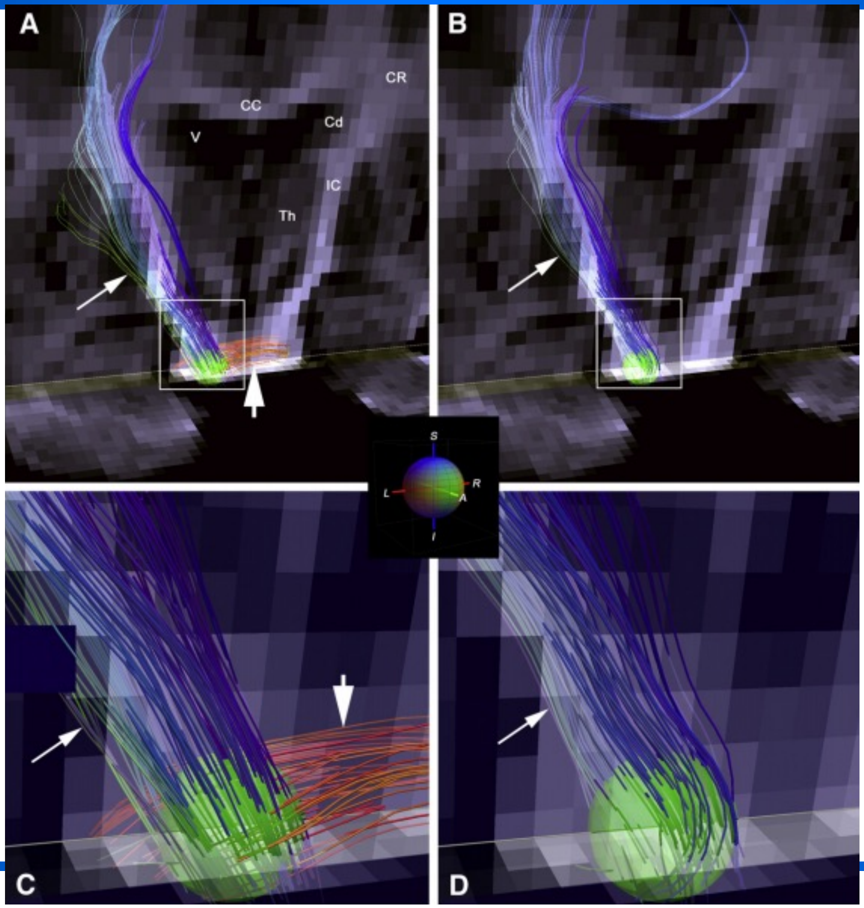
Diffusion Spectrum Imaging



Wedeen V. 2008. Neuroimage

Diffusion Spectrum Imaging

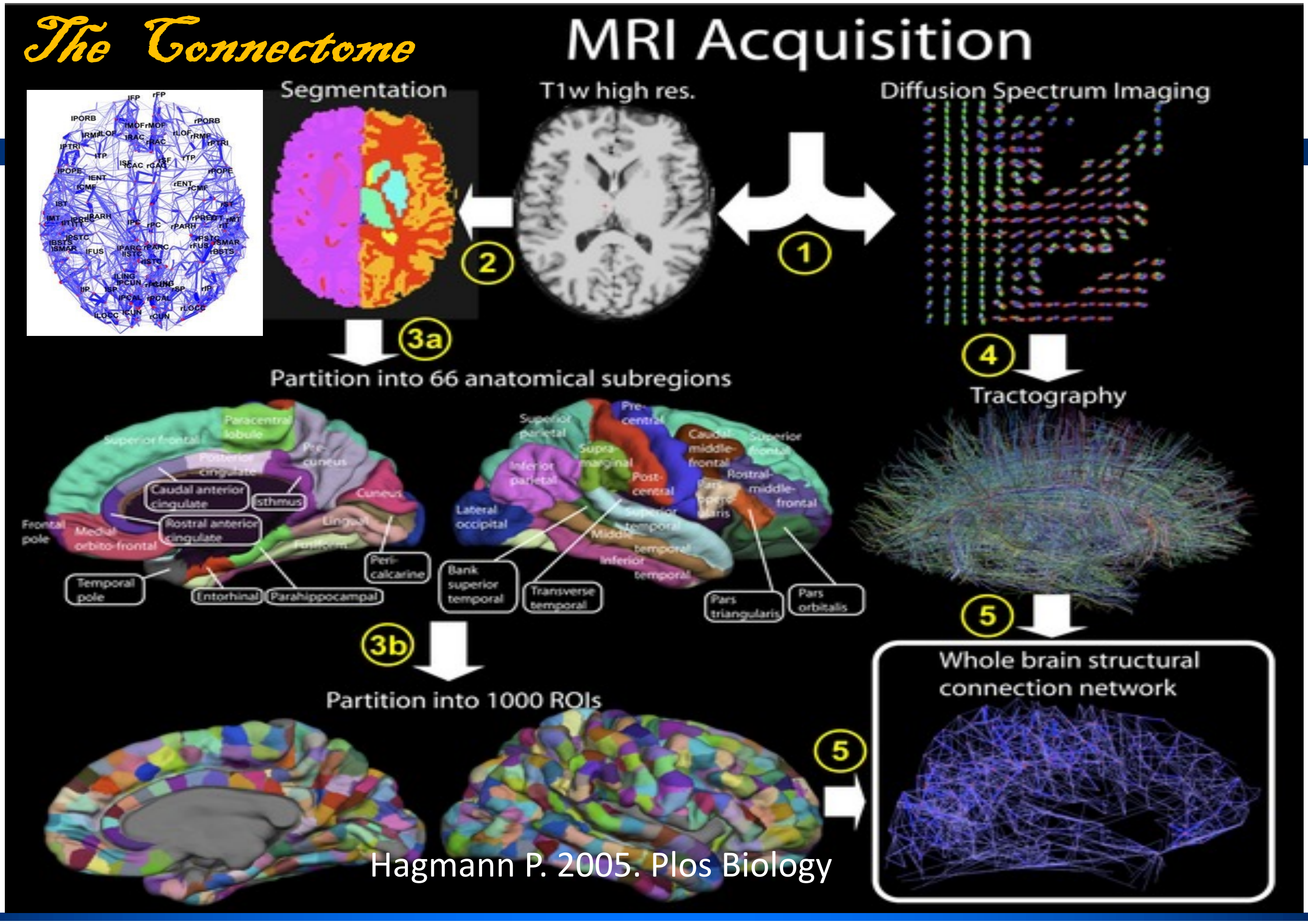


Wedeen V. 2008. Neuroimage

Other Advanced Methods

- Q-Ball Imaging
- Multi-Tensor Model
- High Order Tensor
- …

Tuch D. 2004. MRM
Liu C. 2010. MRM
Schultz T. 2010.
Med Image Comput Comput Assist Interv.


The Connectome
MRI Acquisition
Segmentation
T1w high res.
Diffusion Spectrum Imaging
1
2
3a
Partition into 66 anatomical subregions
4
Tractography
5
Whole brain structural connection network
3b
Partition into 1000 ROIs
5
Hagmann P. 2005. Plos Biology

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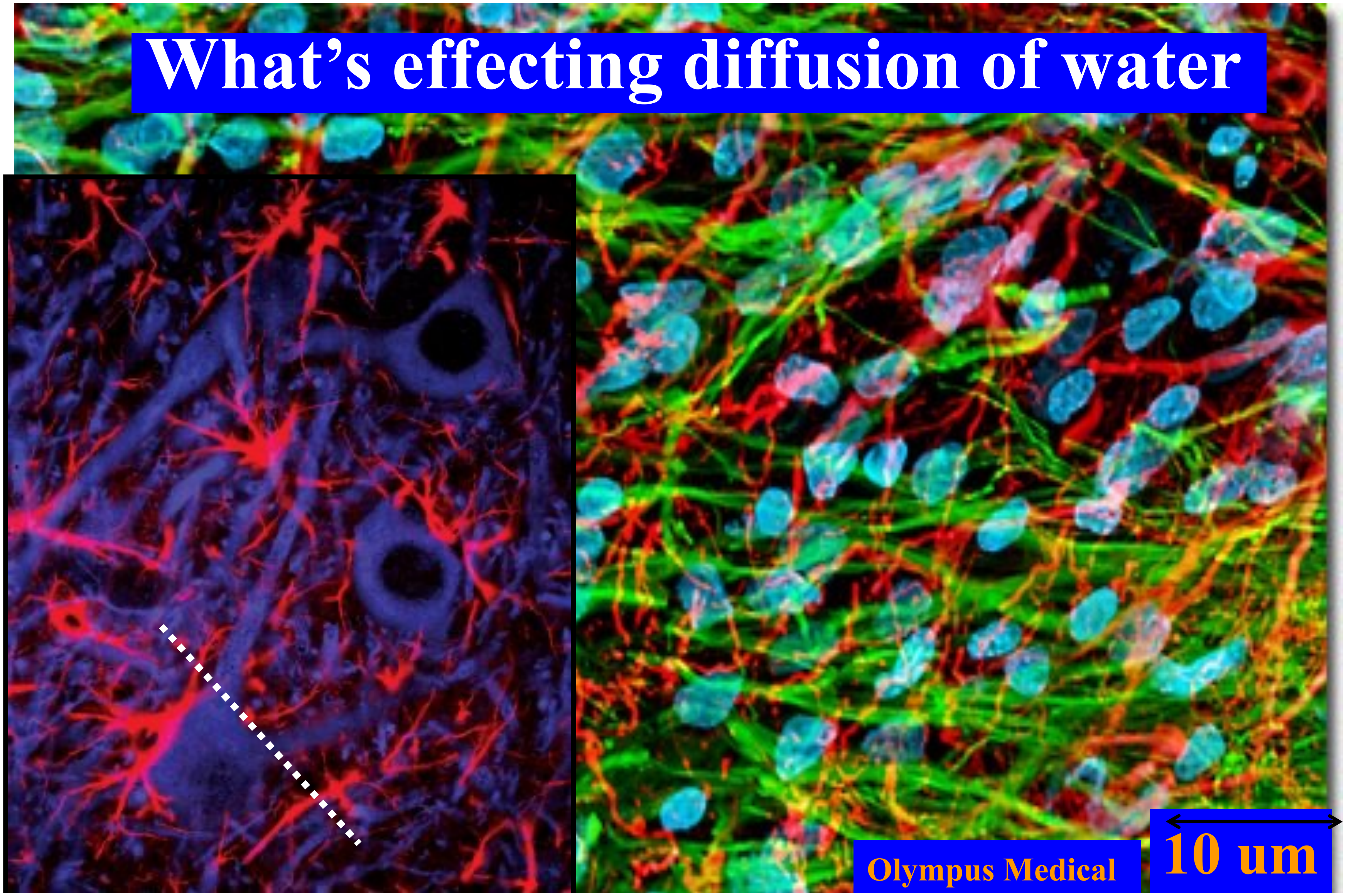
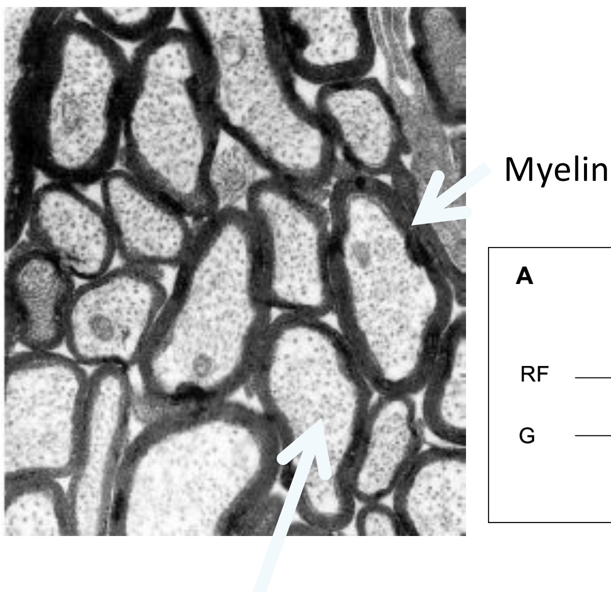
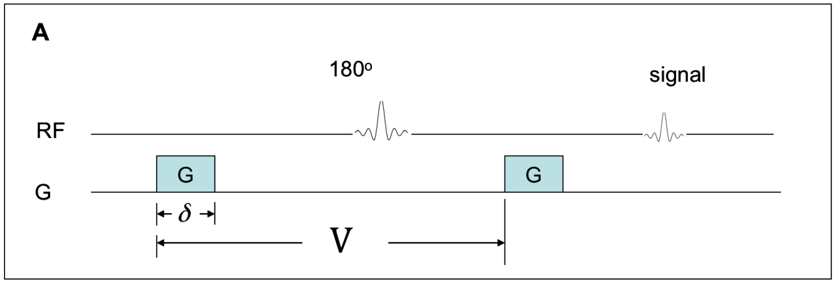
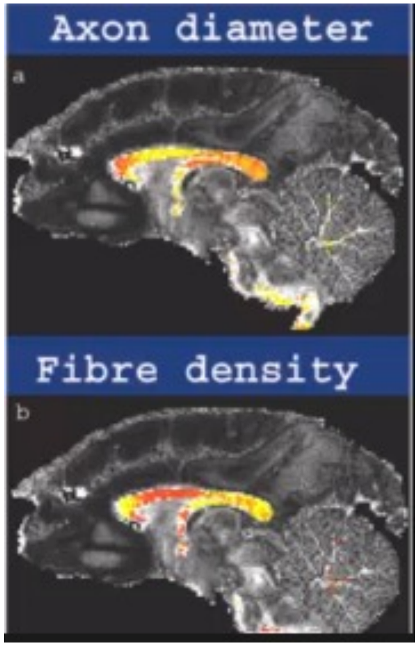
What’s effecting diffusion of water



Microstructure Imaging with Diffusion



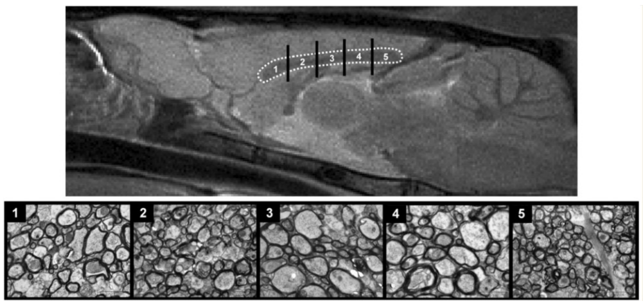


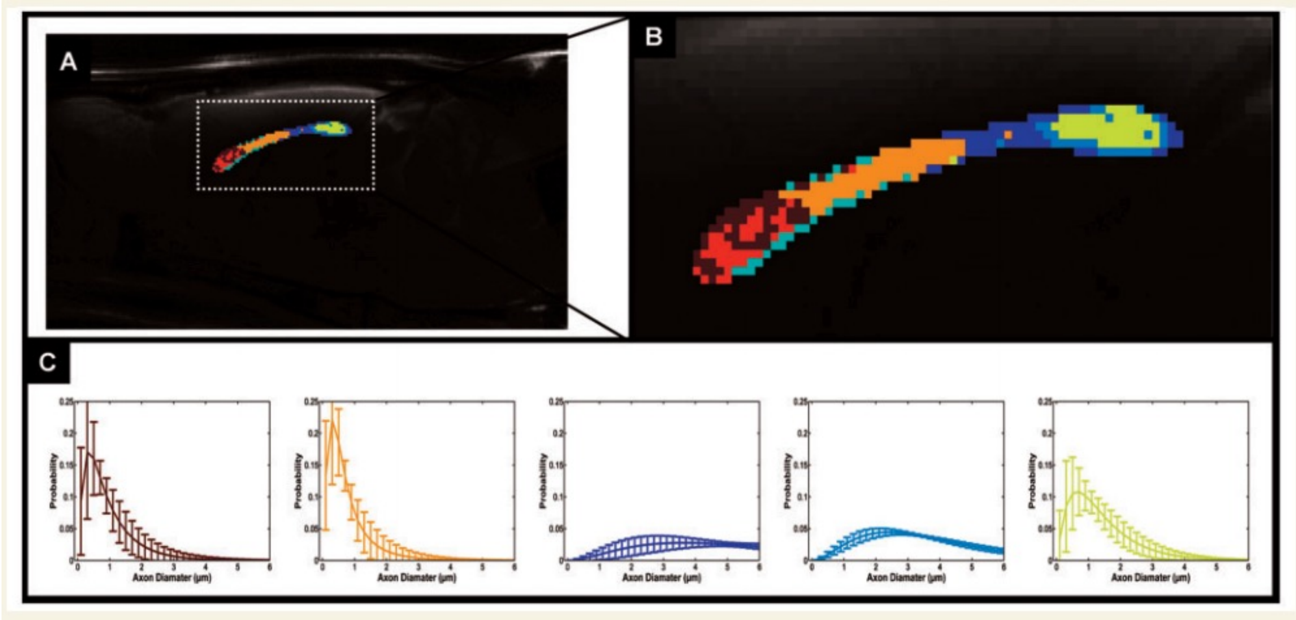


Alexander Neuroimage 2010

AxCalib (Barazany Brain 2009)

δ=3.2ms
Δ={11,20,30,60,100}ms





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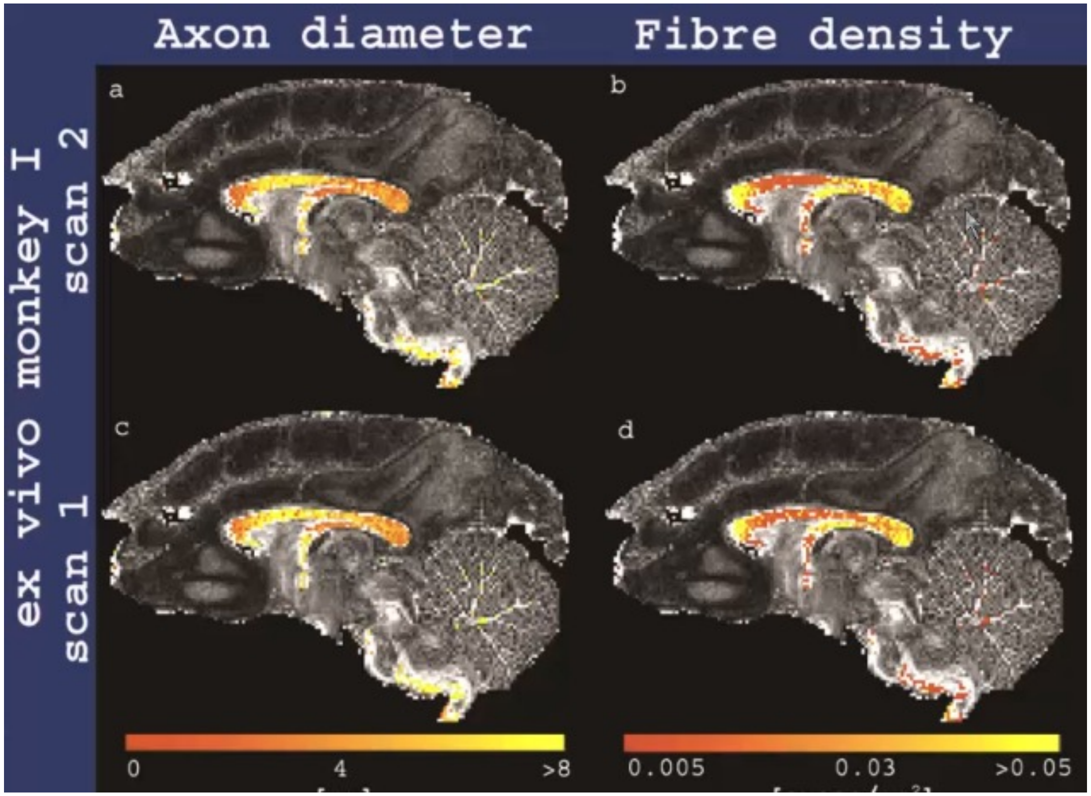
ActiveAx Alexander Neuroimage 2010



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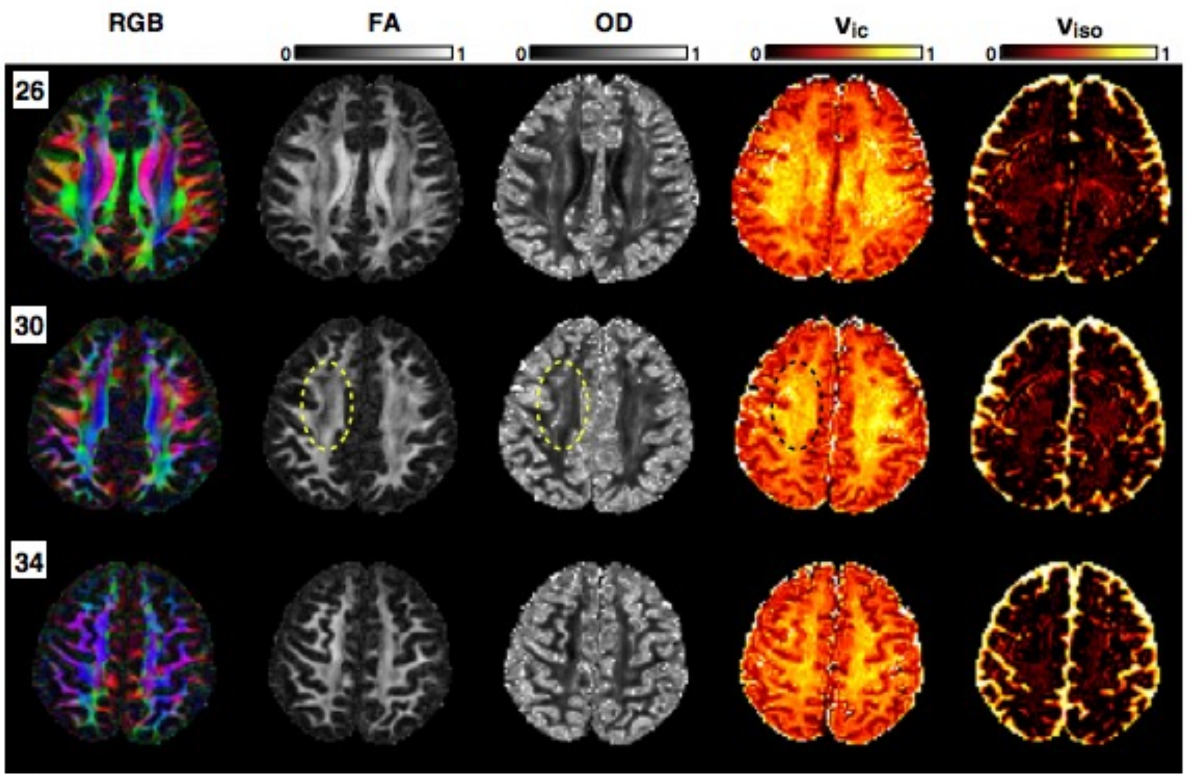

Zhang H. et al. 2012 NeuroImage

Other important topics not covered

- Multi-shot diffusion MRI sequence
- Eddy-current induced distortion
- Double-refocused spin echo for reducing eddy-current distortion
- Other diffusion sensitizing approach
  - Stimulated echo-based diffusion weighting

Thank You

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